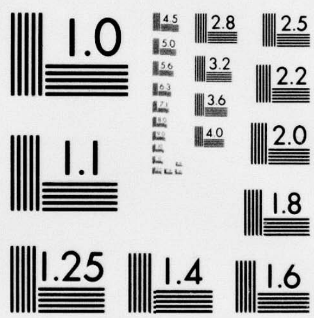
AD-A075 527 ARMY MATERIALS AND MECHANICS RESEARCH CENTER WATERTOWN MA F/G 20/11
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPE--ETC(U)
JUL 79 O L BOWIE , F I BARATTA

UNCLASSIFIED AMMRC-TR-79-46 NL

| OF |

AD
A075527

END
DATE
FILMED
11 -79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AMMRC TR 79-46

(12) B.S.

AD A075527

AD

LEVEL II

ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS

OSCAR L. BOWIE and FRANCIS I. BARATTA
MECHANICS OF MATERIALS DIVISION

July 1979

Approved for public release; distribution unlimited.

ARMY MATERIALS AND MECHANICS RESEARCH CENTER
Watertown, Massachusetts 02172

79 10 22 038

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

Mention of any trade names or manufacturers in this report shall not be construed as advertising nor as an official indorsement or approval of such products or companies by the United States Government.

DISPOSITION INSTRUCTIONS

Destroy this report when it is no longer needed.
Do not return it to the originator.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 14 AMMRC-TR-79-46	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS.		5. TYPE OF REPORT & PERIOD COVERED 9 Final Rep[redacted]rt, 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Oscar L. Bowie [redacted] Francis I. Baratta		8. CONTRACT OR GRANT NUMBER(s) 16
9. PERFORMING ORGANIZATION NAME AND ADDRESS Army Materials and Mechanics Research Center Watertown, Massachusetts 02172 DRXMR-TM		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS D/A Project: 1L162105AH84 AMCMS Code: 612105.H840011
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Materiel Development and Readiness Command, Alexandria, Virginia 22333		12. REPORT DATE 11 Jul[redacted]79 13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) 12 16		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

To be published in the International Journal of Fracture Mechanics.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fracture (mechanics)
Stress intensity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(SEE REVERSE SIDE)

403 105 Gw

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block No. 20

ABSTRACT

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and a center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

1. INTRODUCTION

This paper is concerned with the derivation of a relationship between the net force, the end displacement, and the stress intensity for cracked rectangular specimens with constrained ends. Two cracked-panel configurations are considered: an edge crack and a center crack. The relationship is generally derived using the edge crack case as an example and via a change in appropriate geometry parameters the center crack case is also solved.

2. EDGE CRACK

As an example, in Reference 1 the case of an edge crack in a rectangular panel, Figure 1, was considered with displacement end conditions

$$\begin{aligned} &V = 0,\ U = U_0 \text{ (constant) on } CD \\ &V = 0,\ U = -U_0 \text{ on } AB \end{aligned} \tag{1}$$

The results were normalized with respect to an averaged applied stress σ_A, where

$$\sigma_A = (1/h) \int_D^C \sigma_x dy. \tag{2}$$

In Reference 2, similar results were presented for the center-cracked rectangular panel. Yet if σ_A is regarded as fixed (with respect to L), then $U_0 = U(L)$ and vice versa. If the alternate normalization with respect to U_0 is preferable, then a relationship between U_0, σ_A, and the stress intensity factor K is necessary for the interpretation of the results presented in References 1 and 2.

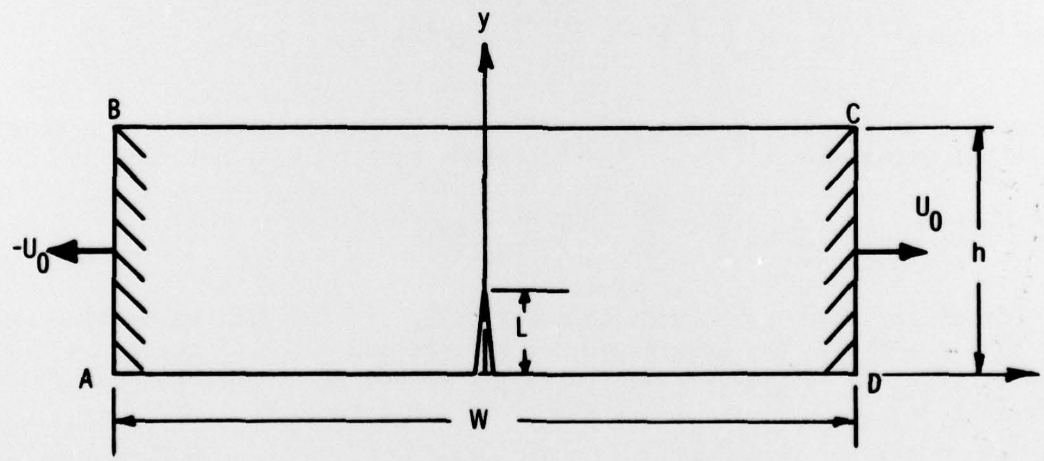


Figure 1. Edge crack in a rectangular panel.

1. BOWIE, O. L., et al. *Solution of Plane Problems of Elasticity Utilizing Partitioning Concepts.* J. Applied Mechanics, v. 95, 1973, p. 767-772.
2. BOWIE, O. L. *Methods of Analysis and Solutions of Crack Problems (Chapter I).* G. C. Sih, ed., Noordhoff International Publishing, Leyden, 1973.

Although the basic results of this paper can be arrived at by compliance arguments, we choose a more interesting approach from an analytical point of view by utilizing an extension of the "weight function" arguments of Bueckner[3] and Rice.[4]

First, we summarize the original weight function argument with Rice's notation as follows: Neglecting body forces and considering only those configurations consistent with Mode I behavior, we assume that a solution has been found for a given geometrical configuration "1", with boundary τ, crack length L, for an applied load with stress vector $\bar{t}^{(1)}(x,y)$, a function of position only. The corresponding stress intensity factor will be denoted by $K^{(1)}(L)$ and displacement vector by $\bar{U}^{(1)}(x,y,L)$. It is assumed that $\bar{t}^{(1)}(x,y)$ is chosen such that $K^{(1)}(L) \neq 0$. Then, for the same geometrical configuration and any load system "2" with an applied stress vector $\bar{t}^{(2)}(x,y)$ consistent, of course, with Mode I behavior,

$$2K^{(1)}(L)\ K^{(2)}(L) = H \int_\tau \bar{t}^{(2)} \cdot \frac{d\bar{U}^{(1)}}{dL}\, d\tau, \tag{3}$$

where $H = E/(1 - \nu^2)$ for plane strain and $H = E$ for plane stress. In Equation 3, it is assumed that $\sqrt{\pi}$ is included in the definitions of $K^{(1)}(L)$ and $K^{(2)}(L)$, i.e., in the vicinity of the crack tip

$$\sigma \approx K(L)\ (2\pi r)^{-1/2}. \tag{4}$$

For the problems of end constraints, the stress vector on the ends is a function of the parameter L as well as position, thus Equation 3 is not applicable. The Reciprocal Theorem of Betti and Rayleigh used in deriving Equation 3 can be easily extended to the case when $\bar{t}^{(1)}(x,y,L)$ is a function of both position and the parameter L. In fact,

$$2K^{(1)}(L)\ K^{(2)}(L) = H \int_\tau \left\{ \bar{t}^{(2)} \cdot \frac{d\bar{U}^{(1)}}{dL} - \bar{U}^{(2)} \cdot \frac{d\bar{t}^{(1)}}{dL} \right\} d\tau, \tag{5}$$

which reduces to Equation 3 when $d\bar{t}^{(1)}/dL = 0$. We shall be concerned primarily with the situation when $\bar{t}^{(1)} = \bar{t}^{(2)} = \bar{t}$. Then, with obvious notation

$$2\left[K(L)\right]^2 = H \int_\tau \left\{ \bar{t} \cdot \frac{d\bar{U}}{dL} - \bar{U} \cdot \frac{d\bar{t}}{dL} \right\} d\tau. \tag{6}$$

Consider now the displacement boundary value problem defined by Equation 1. Assume that a solution has been found for a fixed end force, i.e., σ_A is independent of L. Then U_0 must be considered as a function of L. Utilizing symmetry and Equation 6,

$$H \int_0^h \left\{ \sigma_x \frac{dU_0}{dL} - U_0 \frac{d\sigma_x}{dL} \right\} dy = \left[K^{(1)}(L) \right]^2 \tag{7}$$

3. BUECKNER, H. F. *A Novel Principle for the Computation of Stress Intensity Factors.* Z. Angew. Math. Mech., v. 50, 1970, p. 529-546.
4. RICE, J. R. *Some Remarks on Elastic Crack-Tip Stress Fields.* International Journal Solids Structures, v. 8, no. 6, June 1972, p. 751-758.

or

$$H \frac{dU_0}{dL} \int_0^h \sigma_x dy - HU_0 \frac{d}{dL} \left\{ \int_0^h \sigma_x dy \right\} = \left[K^{(1)}(L) \right]^2,$$

$$Hh\sigma_A \frac{dU_0}{dL} = \left[K^{(1)}(L) \right]^2 \tag{8}$$

$$Hh\sigma_A\, U_0(L) = \int_0^L \left[K^{(1)}(\xi) \right]^2 d\xi + C_1,$$

where the constant of integration C_1 must be chosen such that Equation 8 is consistent at $L = 0$.

If, on the other hand, a solution has been found normalized with respect to a constant value of U_0, Equation 6 becomes

$$-HU_0 \int_0^h \frac{d\sigma_x}{dL}\, dy = \left[K^{(2)}(L) \right]^2, \tag{9}$$

or

$$HU_0 h\, \sigma_A(L) = - \int_0^L \left[K^{(2)}(\xi) \right]^2 d\xi + C_1', \tag{10}$$

where for a fixed value of U_0, $\sigma_A(L)$ is a decreasing function of L. In fact, $\sigma_A(h) = 0$ and

$$C_1' = \int_0^h \left[K^{(2)}(\xi) \right]^2 d\xi. \tag{11}$$

The constants of integration C_1 and C_1' cannot be derived in a direct rigorous manner owing to an unknown distribution of shear stresses on the ends. For practical purposes, however, its value can be established by the following argument: Consider the two load systems in Figure 2. We choose

$$\sigma_x^{(1)} = \sigma_A = h^{-1} \int_{-h/2}^{h/2} \sigma_x^{(2)} dy, \tag{12}$$

and

$$\sigma_y^{(1)} = [(H/2\mu) - 1]\, \sigma_A, \tag{13}$$

where μ is the shear modulus. For this load system, the displacement V in system "1" vanishes and

$$U^{(1)} = [\mu^{-1} - H/4\mu^2]\, \sigma_A x. \tag{14}$$

Applying the Reciprocal Theorem,

$$U_0 h \sigma_A = \sigma_A \left[\mu^{-1} - H/4\mu^2 \right] \frac{hW}{2} - [(H/2\mu) - 1]\, \sigma_A \int_{-W/2}^{W/2} V^{(2)} dx. \tag{15}$$

Although we do not know $V^{(2)}$ explicitly it is evidently a measure of the vertical contraction of System "2". It is evident physically that if we assume $V^{(2)} = 0$, a lower bound on U_0 is obtained from Equation 15,

$$U_0 h\sigma_A \geq \sigma_A \left[\mu^{-1} - H/4\mu^2\right] \frac{hW}{2}, \tag{16}$$

with equality for $W/h \ll 1$.

On the other hand, a reasonably accurate upper bound can be found by assuming a uniform lateral contraction $V^{(2)} = [H^{-1} - 1/2\mu]\sigma_A h/2$, which leads to

$$U_0 h\sigma_A \leq \sigma_A^2\, Wh/2H. \tag{17}$$

On the basis of numerical results for $W/h = 1$, the equality sign in Equation 17 can be taken with satisfactory accuracy. On physical grounds, the accuracy of this approximation increases with increasing W/h. Thus,

$$C_1 \simeq \sigma_A^2\, Wh/2, \quad W/h > 1. \tag{18}$$

By introducing the function $\left[M_{\sigma_A}(L/h)\right]$ where

$$K(L/h) = \left[M_{\sigma_A}(L/h)\right] \sigma_A (\pi L)^{\frac{1}{2}}. \tag{19}$$

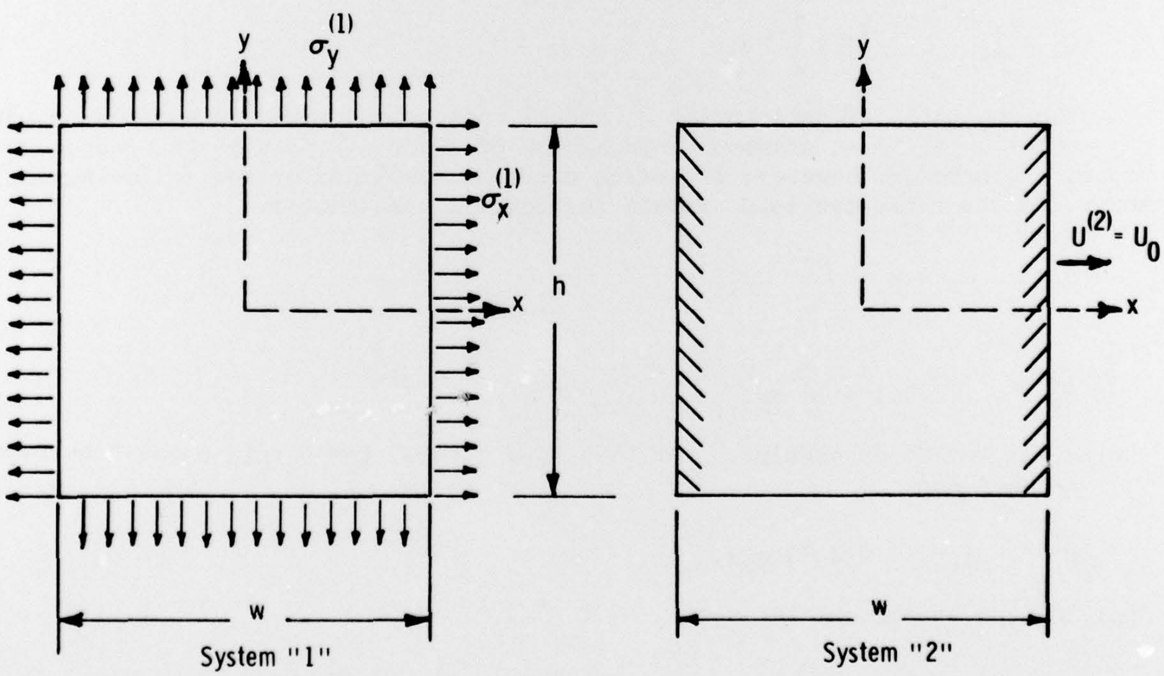


Figure 2. Load systems "1" and "2".

Equations 8 and 18 can be written in this notation as

$$\frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{1/2}} = \frac{\left[M_{\sigma_A}(L/h)\right]}{1 + 2\pi(h/W)\int_0^{L/h}\left[M_{\sigma_A}(L/h)\right]^2 (L/h)\ d(L/h)} . \tag{20}$$

Regarding the constant C_1', when U_0 is kept fixed, we have from the geometry of Figure 1 that when $L = 0$

$$C' = HhU_0\sigma_A(0) = HhU_0\left(\frac{2HU_0}{W}\right) \tag{21}$$

or

$$C_1' = 2H^2U_0^2\ (h/W) .$$

Similarly, if the function $\left[M_{U_0}(L/h)\right]$ is defined as

$$K(L/h) = \left[M_{U_0}(L/h)\right] H\left(\frac{2U_0}{W}\right)(\pi L)^{1/2} \tag{22}$$

Equations 10 and 21 can be written as

$$\frac{K}{\sigma_A(\pi L)^{1/2}} = \frac{M_{U_0}(L/h)}{1 - 2\pi(h/W)\int_0^{L/h}\left[M_{U_0}(L/h)\right]^2 (L/h)\ d(L/h)} . \tag{23}$$

Note when $h/W \to 0$ or $L/h \to 0$ from Equation 20 or 23 we have

$$\left[M_{U_0}(L/h)\right] \equiv \left[M_{\sigma_A}(L/h)\right], \tag{24}$$

as expected.

The validity of the approximations to C_1 and C_1' in Equations 18 and 21 is implemented through the use of Equations 20 and 23 and illustrated in Table 1 using results of Freese.* His data corresponds to the plane stress constrained end problem with $h/W = 0.983$ and $\nu = 0.35$.

Equation 20 was used to obtain $\left[M_{U_0}(L/h)\right]$ from the data given in Reference 1, where $\left[M_{\sigma_A}(L/h)\right]$ was given for the loading case described by Figure 1, for $\nu = 1/4$. These results are shown in Table 2.

Shown in Table 3, for comparison, are some other known results for this problem from A. S. Kobayashi and S. Mall reported in Reference 5, for $W/h = 3$. It is seen that the greatest difference between the results reported here and those given in Reference 5 is approximately 6% occurring at $L/h = 0.70$.

*FREESE, C. E., Army Materials and Mechanics Research Center, private communication, 1975.

5. QUACKENBUSH, C. L., and FRECHETTE, V. D. *Crack-Front Curvature and Glass Slow Fracture.* J. Am. Ceram. Soc., v. 61, nos. 9-10, September-October 1978, p. 402-406.

Table 1. NORMALIZED STRESS INTENSITY FACTORS

	$\left[M_{U_0}(L/h)\right] = \frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{1/2}}$		$\left[M_{\sigma_A}(L/h)\right] = \frac{K}{\sigma(\pi L)^{1/2}}$	
L/h	Freese Data	Eq. 20	Freese Data	Eq. 23
0				
0.10	1.03	1.00	1.03	1.03
.20	0.86	0.85	0.95	0.95
.30	.74	.73	.92	.92
.40	.67	.65	.93	.93
.50	.59	.58	.98	.98
.60	.54	.53	1.09	1.09
.70	.50	.49	1.28	1.28
.80	.46	.46	1.63	1.63

Table 2. NORMALIZED STRESS INTENSITY FACTOR FOR AN EDGE CRACK IN A RECTANGULAR PANEL

$$\left[M_{U_0}(L/h)\right] = \frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{1/2}}$$

$\nu = 1/4$

L/h \ W/h	1	2	3	4	8	12	16
0							
0.05	1.01	1.11	1.13	1.13	1.13	1.13	1.13
.10	0.97	1.10	1.14	1.15	1.16	1.18	1.18
.20	.85	1.06	1.14	1.20	1.27	1.30	1.32
.30	.73	0.99	1.14	1.23	1.41	1.49	1.53
.40	.64	.91	1.10	1.24	1.55	1.70	1.79
.50	.58	.83	1.04	1.21	1.65	1.90	2.07
.60	.53	.75	0.95	1.12	1.67	2.04	2.32
.70	.49	.69	.86	1.02	1.58	2.03	2.40

Table 3. COMPARISON OF $\left[M_{U_0}(L/h)\right]$ WHEN W/h = 3

L/h	From Eq. 20 and Ref. 1 $\nu = 1/4$	From Ref. 5 $\nu = 0.23$	Difference (%)
0			
0.05	1.13	1.11	-2
.10	1.14	1.11	-3
.20	1.14	1.11	-3
.30	1.14	1.09	-4
.40	1.10	1.05	-5
.50	1.04	0.99	-5
.60	0.95	.90	-5
.70	.86	.81	-6

3. CENTRAL CRACK

When we apply the Rice type of argument to the central crack shown in Figure 3, we fix end A and consider a perturbation of the end B. Equation 7 holds with the obvious change in notation

$$H \int_{-h/2}^{h/2} \left\{ \sigma_y \frac{dU_0}{da} - \frac{U_0 d\sigma_y}{da} \right\} dx = [K(a)]^2, \tag{25}$$

where K(a) is the stress intensity at B. Now suppose we introduce the conventional notation a = 2L, then K(a) is equivalent to the conventional K(L). Note, however, that now

$$H \int_{-h/2}^{h/2} \left\{ \sigma_y \frac{dU_0}{d(2L)} - U_0 \frac{d\sigma_y}{d(2L)} \right\} dx = [K(L)]^2, \tag{26}$$

leading to

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

AD ________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

AD ________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

AD ________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

AD ________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

AD ______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

AD ______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

AD ______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
ON THE LOAD-DISPLACEMENT RELATION FOR A CRACKED RECTANGULAR SPECIMEN WITH CONSTRAINED ENDS –
Oscar L. Bowie and Francis I. Baratta

AD ______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Fracture (mechanics)
Stress intensity

Technical Report AMMRC TR 79-46, July 1979, 10 pp – illus-tables, D/A Project 1L162105AH84, AMCMS Code 612105.H840011

Relationships are presented between the net force, the end displacement, and the stress intensity factor for cracked rectangular panels subjected to uniform displacement. Two configurations are considered: an edge-cracked panel and center-cracked panel. If the stress intensity factor is already known in terms of the normal displacement, then the resulting formulation allows the determination of that stress intensity in terms of the applied load and vice versa.

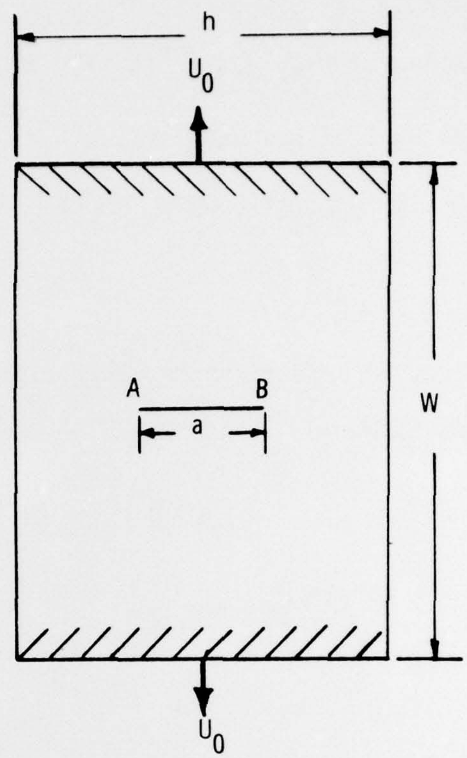


Figure 3. Center crack in a rectangular panel.

$$Hh\sigma_A \frac{dU}{2dL} = [K(L)]^2. \tag{27}$$

Thus Equation 8 now becomes

$$Hh\sigma_A U_0(L) = 2 \int_0^L [K(\xi)]^2 \, d\xi + C_1. \tag{28}$$

The constant $C_1 \approx \sigma_A^2 \, Wh/2$, $W/h > 1$ so that

$$Hh\sigma_A U_0(L) = \sigma_A^2 \, Wh/2 + 2 \int_0^L [K(\xi)]^2 \, d\xi + C_1. \tag{29}$$

Substituting C_1 into Equation 29 and proceeding as before, but with

$$K = \left[M_{\sigma_A}(2L/h)\right] \sigma_A (\pi L)^{\frac{1}{2}}, \tag{30}$$

for the center-cracked panel, we can rewrite Equation 29 as

$$\frac{K}{H\left(\frac{2U_0}{W}\right)(\pi L)^{\frac{1}{2}}} = \frac{\left[M_{\sigma A}(2L/h)\right]}{1 + \pi(h/W)\int_0^{2L/h}\left[M_{\sigma_A}(2L/h)\right]^2 (2L/h)\,d(2L/h)}. \tag{31}$$

Regarding the constant C_1' for the center crack case, but allowing for a factor of 2, i.e.,

$$HU_0 h\sigma_A(L) = C_1' - 2\int_0^L [K(\xi)]^2 \, d\xi, \tag{32}$$

and when U_0 is fixed but letting $L = 0$ from Figure 3, we have

$$C_1' = Hh\ U_0\sigma_A(0) = Hh\ U_0 \left(\frac{2HU_0}{W}\right)$$

or

$$C_1' = 2H^2U_0^2\ (h/W),$$

as determined before for the edge crack case.

Substituting C_1' into Equation 32 and defining K for the center-cracked panel as

$$K = \left[M_{U_0}(2L/h)\right] H \left(\frac{2U_0}{W}\right) (\pi L)^{\frac{1}{2}}, \tag{33}$$

we finally obtain

$$\frac{K}{\sigma(L)(\pi L)^{\frac{1}{2}}} = \frac{\left[M_{U_0}(2L/h)\right]}{1 - \pi(h/W)\int_0^{2L/h}\left[M_{U_0}(2L/h)\right]^2 (2L/h)\,d(2L/h)}\ . \tag{34}$$

Again as expected, it is noted that when $h/W \rightarrow 0$ or $2L/h \rightarrow 0$, Equations 31 and 34 reduce to

$$\left[M_{U_0}(2L/h)\right] \equiv \left[M_{\sigma_A}(2L/h)\right]. \tag{35}$$

4. CLOSING COMMENTS

Equation 24 or 35 can serve as a check on the accuracy of known results when $h/W \rightarrow 0$ or $L/h \rightarrow 0$ for the edge crack, and $2L/h \rightarrow 0$ for the center-cracked panel subjected to uniform normal displacement. Further, Equation 11 can be used to provide a check on the limiting end point, i.e., $L/h = 1.0$ for the edge crack or $2L/h = 1.0$ for the center crack, if $\left[M_{U_0}(L/h)\right]$ or $\left[M_{U_0}(2L/h)\right]$ are known.

It is expected that the method outlined in sections 2 and 3, although specifically applicable to an edge- or center-cracked panel, can be generally applied to other cracked-body configurations subjected to displacement type loadings. The results of the method described here are particularly useful if the stress intensity factor is known only as a function of the applied load and it is desired as a function of the normal displacement, or vice versa.

DISTRIBUTION LIST

No. of Copies	To
1	Office of the Under Secretary of Defense for Research and Engineering, The Pentagon, Washington, D.C. 20301
12	Commander, Defense Documentation Center, Cameron Station, Building 5, 5010 Duke Street, Alexandria, Virginia 22314
1	Metals and Ceramics Information Center, Battelle Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201
	Deputy Chief of Staff, Research, Development, and Acquisition, Headquarters Department of the Army, Washington, D. C. 20310
1	ATTN: DAMA-ARZ
	Commander, Army Research Office, P. O. Box 12211, Research Triangle Park, North Carolina 27709
1	ATTN: Information Processing Office
1	Dr. F. W. Schmiedeshoff
	Commander, U. S. Army Materiel Development and Readiness Command, 5001 Eisenhower Avenue, Alexandria, Virginia 22333
1	ATTN: DRCLDC, Mr. R. Zentner
	Commander, U. S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, Maryland 21005
1	ATTN: DRXSY-MP, H. Cohen
	Commander, U. S. Army Communications Research and Development Command, Fort Monmouth, New Jersey 07703
1	ATTN: DELSD-L
1	DELSD-E
	Commander, U. S. Army Missile Research and Development Command, Redstone Arsenal, Alabama 35809
1	ATTN: DRDMI-RKK, Mr. C. Martens, Bldg. 7120
	Commander, U. S. Army Natick Research and Development Command, Natick, Massachusetts 01760
1	ATTN: Technical Library
1	Dr. E. W. Ross
1	DRDNA-UE, Dr. L. A. McClaine
	Commander, U. S. Army Satellite Communications Agency, Fort Monmouth, New Jersey 07703
1	ATTN: Technical Document Center
	Commander, U. S. Army Tank-Automotive Research and Development Command, Warren, Michigan 48090
1	ATTN: DRDTA-RKA
1	DRDTA-UL, Technical Library
	Commander, U. S. Army Armament Research and Development Command, Dover, New Jersey 07801
2	ATTN: Technical Library
1	DRDAR-SCM, J. D. Corrie
1	Dr. J. Fraiser
1	Mr. Harry E. Pebly, Jr., PLASTEC, Director
	Commander, White Sands Missile Range, New Mexico 88002
1	ATTN: STEWS-WS-VT
	Commander, U. S. Army Armament Research and Development Command, Aberdeen Proving Ground, Maryland 21010
1	ATTN: DRDAR-QAC-E
	Commander, U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland 21005
1	ATTN: Dr. R. Vitali
1	Dr. G. L. Filbey
1	Dr. W. Gillich
1	DRDAR-TSB-S (STINFO)
	Commander, Harry Diamond Laboratories, 2800 Powder Mill Road, Adelphi, Maryland 20783
1	ATTN: Technical Information Office

No. of Copies	To
	Commander, Picatinny Arsenal, Dover, New Jersey 07801
1	ATTN: Mr. J. Pearson
1	G. Randers-Pehrson
1	Mr. A. Garcia
1	SARPA-RT-S
	Commander, Redstone Scientific Information Center, U. S. Army Missile Research and Development Command, Redstone Arsenal, Alabama 35809
1	ATTN: DRDMI-TB
	Commander, Watervliet Arsenal, Watervliet, New York 12189
1	ATTN: Dr. T. Davidson
1	Mr. D. P. Kendall
1	Mr. J. F. Throop
	Commander, U. S. Army Foreign Science and Technology Center, 220 7th Street, N. E., Charlottesville, Virginia 22901
1	ATTN: Mr. Marley, Military Tech
	Chief, Benet Weapons Laboratory, LCWSL, USA ARRADCOM, Watervliet Arsenal, Watervliet, New York 12189
1	ATTN: DRDAR-LCB-TL
	Director, Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia 23604
1	ATTN: Mr. J. Robinson, DAVDL-E-MOS (AVRADCOM)
	U. S. Army Aviation Training Library, Fort Rucker, Alabama 36360
1	ATTN: Building 5906-5907
	Commander, U. S. Army Agency for Aviation Safety, Fort Rucker, Alabama 36362
1	ATTN: Librarian, Bldg. 4905
	Commander, USACDC Air Defense Agency, Fort Bliss, Texas 79916
1	ATTN: Technical Library
	Commander, U. S. Army Engineer School, Fort Belvoir, Virginia 22060
1	ATTN: Library
	Commander, U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi 39180
1	ATTN: Research Center Library
	Commander, U. S. Army Mobility Equipment Research and Development Center, Fort Belvoir, Virginia 22060
1	ATTN: DRDME-MW, Dr. J. W. Bond
	Commander, Naval Air Engineering Center, Lakehurst, New Jersey 08733
1	ATTN: Technical Library, Code 1115
	David Taylor Naval Ship Research and Development Laboratory, Annapolis, Maryland 21402
1	ATTN: Dr. H. P. Chu
	Naval Research Laboratory, Washington, D.C. 20375
1	ATTN: C. D. Beachem, Head, Adv. Mat'ls Tech Br. (Code 6310)
1	Dr. J. M. Krafft - Code 8430
1	E. A. Lange
1	Dr. P. P. Puzak
1	R. J. Sanford - Code 8436
1	A. M. Sullivan
1	R. W. Rice
1	S. W. Freiman
1	Dr. Jim C. I. Chang

No. of Copies	To
	Chief of Naval Research, Arlington, Virginia 22217
1	ATTN: Code 471
	Naval Weapons Laboratory, Washington, D.C. 20390
1	ATTN: H. W. Romine, Mail Stop 103
1	Ship Research Committee, Maritime Transportation Research Board, National Research Council, 2101 Constitution Avenue, N. W., Washington, D.C. 20418
	Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433
2	ATTN: AFML (MXE), E. Morrissey
1	AFML (LC)
1	AFML (LLP), D. M. Forney, Jr.
1	AFML (LNC), T. J. Reinhart
1	AFML (MBC), Mr. Stanley Schulman
1	AFFDL (FB), Dr. J. C. Halpin
	Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433
1	ATTN: AFFDL (FBS), C. Wallace
1	AFFDL (FBE), G. D. Sendeckyj
	National Aeronautics and Space Administration, Washington, D.C. 20546
1	ATTN: Mr. B. G. Achhammer
1	Mr. G. C. Deutsch - Code RW
	National Aeronautics and Space Administration, Marshall Space Flight Center, Huntsville, Alabama 35812
1	ATTN: R. J. Schwinghamer, EH01, Dir., M&P Lab
1	Mr. W. A. Wilson, EH41, Bldg. 4612
	National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia 23665
1	ATTN: Mr. H. F. Hardrath, Mail Stop 188M
1	Mr. R. Foye, Mail Stop 188A
	National Aeronautics and Space Administration, Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44135
1	ATTN: Dr. J. E. Srawley, Mail Stop 105-1
1	Mr. W. F. Brown, Jr.
1	Mr. M. H. Hirschberg, Head, Fatigue Research Section, Mail Stop 49-1
1	National Bureau of Standards, U. S. Department of Commerce, Washington, D.C. 20234
1	ATTN: Mr. J. A. Bennett
1	Mechanical Properties Data Center, Belfour Stulen Inc., 13917 W. Bay Shore Drive, Traverse City, Michigan 49684
	Midwest Research Institute, 425 Coker Boulevard, Kansas City, Missouri 64110
1	ATTN: Mr. G. Gross
1	Mr. J. G. Kaufman, Alcoa Research Laboratories, New Kensington, Pennsylvania 15068
1	Mr. P. N. Randall, TRW Systems Group - O-1/2210, One Space Park, Redondo Beach, California 90278
1	Dr. E. A. Steigerwald, TRW Metals Division, P. O. Box 250, Minerva, Ohio 44657
1	Dr. George R. Irwin, Department of Mechanical Engineering, University of Maryland, College Park, Maryland 20742
1	Mr. W. A. Van der Sluys, Research Center, Babcock and Wilcox, Alliance, Ohio 44601
1	Mr. B. M. Wundt, 2346 Shirl Lane, Schenectady, New York 12309

No. of Copies	To
	Battelle Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201
1	ATTN: Mr. J. Campbell
1	Dr. G. T. Hahn
1	R. G. Hoagland, Metal Science Group
1	Dr. E. Rybicki
	General Electric Company, Schenectady, New York 12010
1	ATTN: Mr. A. J. Brothers, Materials & Processes Laboratory
	General Electric Company, Schenectady, New York 12309
1	ATTN: Mr. S. Yukawa, Metallurgy Unit
1	Mr. E. E. Zwicky, Jr.
	General Electric Company, Knolls Atomic Power Laboratory, P. O. Box 1072, Schenectady, New York 12301
1	ATTN: Mr. F. J. Mehringer
1	Dr. L. F. Coffin, Room 1C41-K1, Corp. R&D, General Electric Company, P. O. Box 8, Schenectady, New York 12301
	United States Steel Corporation, Monroeville, Pennsylvania 15146
1	ATTN: Mr. S. R. Novak
1	Dr. A. K. Shoemaker, Research Laboratory, Mail Stop 78
	Westinghouse Electric Company, Bettis Atomic Power Laboratory, P. O. Box 109, West Mifflin, Pennsylvania 15122
1	ATTN: Mr. R. G. Hoppe, Manager
	Westinghouse Research and Development Center, 1310 Beulah Road, Pittsburgh, Pennsylvania 15235
1	ATTN: Mr. E. T. Wessel
1	Mr. M. J. Manjoine
	Brown University, Providence, Rhode Island 02912
1	ATTN: Prof. J. R. Rice
1	Prof. W. N. Findley, Division of Engineering, Box D
1	Prof. P. C. Paris
	Carnegie-Mellon University, Department of Mechanical Engineering, Schenley Park, Pittsburgh, Pennsylvania 15213
1	ATTN: Dr. J. L. Swedlow
1	Prof. J. D. Lubahn, Colorado School of Mines, Golden, Colorado 80401
1	Prof. J. Dvorak, Civil Engineering Department, Duke University, Durham, North Carolina 27706
	George Washington University, School of Engineering and Applied Sciences, Washington, D.C. 20052
1	ATTN: Dr. H. Liebowitz
	Lehigh University, Bethlehem, Pennsylvania 18015
1	ATTN: Prof. G. C. Sih
1	Prof. R. Roberts
1	Prof. R. P. Wei
1	Prof. F. Erodgan
	Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
1	ATTN: Prof. B. L. Averbach, Materials Center, 13-5082
1	Prof. F. A. McClintock, Room 1-304
1	Prof. R. M. Pelloux
1	Prof. T. H. H. Pian, Department of Aeronautics and Astronautics
1	Prof. A. S. Argon, Room 1-306
	Syracuse University, Department of Chemical Engineering and Metallurgy, 409 Link Hall, Syracuse, New York 13210
1	ATTN: Mr. H. W. Liu
1	Dr. V. Weiss, Metallurgical Research Labs., Bldg. D-6

No. of Copies	To
1	Prof. E. R. Parker, Department of Materials Science and Engineering, University of California, Berkeley, California 94700
1	Prof. W. Goldsmith, Department of Mechanical Engineering, University of California, Berkeley, California 94720
	University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
1	ATTN: Dr. R. Karpp
1	Prof. A. J. McEvily, Metallurgy Department U-136, University of Connecticut, Storrs, Connecticut 06268
1	Prof. D. Drucker, Dean of School of Engineering, University of Illinois, Champaign, Illinois 61820
	University of Illinois, Urbana, Illinois 61801
1	ATTN: Prof. T. J. Dolan, Department of Theoretical and Applied Mechanics
1	Prof. J. Morrow, 321 Talbot Laboratory
1	Mr. G. M. Sinclair, Department of Theoretical and Applied Mechanics
1	Prof. R. I. Stephens, Materials Engineering Division, University of Iowa, Iowa City, Iowa 52242
1	Prof. D. K. Felbeck, Department of Mechanical Engineering, University of Michigan, 2046 East Engineering, Ann Arbor, Michigan 48109
1	Dr. M. L. Williams, Dean of Engineering, 240 Benedum Hall, University of Pittsburgh, Pittsburgh, Pennsylvania 15260
1	Prof. A. Kobayashi, Department of Mechanical Engineering, FU-10, University of Washington, Seattle, Washington 98195
	State University of New York at Stony Brook, Stony Brook, New York 11790
1	ATTN: Prof. Fu-Pen Chiang, Department of Mechanics
1	Dr. Robert S. Shane, Shane Associates, Inc., 7821 Carrleigh Parkway, Springfield, Virginia 22152
	Director, Army Materials and Mechanics Research Center, Watertown, Massachusetts 02172
2	ATTN: DRXMR-PL
1	DRXMR-WD
2	Authors